

Coupled dynamics of an atom and an optomechanical cavity

X. X. Yi, H. Y. Sun, and L. C. Wang
*School of Physics and Optoelectronic Technology,
Dalian University of Technology, Dalian 116024, China*

We consider the motion of the end mirror of a cavity inside which a two-level atom trapped. The fast vibrating mirror induces nonlinear couplings between the cavity field and the atom. We analyze this optical effect by showing the population of the atom in its internal degrees of freedom as a function of time. On the other side, fast atom-field variables result in an additional potential for the atomic center-of-mass motion and the mirror vibration, leading to entanglement in the motion and the vibration. The entanglement has been numerically simulated and discussed.

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I. INTRODUCTION

Optomechanical cavities [1, 2, 3, 4, 5, 6] in which electromagnetic degrees of freedom couple to the mechanical motion of mesoscopic or macroscopic mirrors are promising candidates for studying the transition of a macroscopic degree of freedom from the classical to the quantum regime. These systems also offer the prospect of technological use, for example the single molecule detection[7], the gravitational wave detection[8] and the possible new quantum information processing devices[9]. In these radiation-pressure-driven devices, the number of photons trapped inside the cavity is a key variable, since the radiation pressure on the mirror is proportional to the photon number. Several strategies to increase this optomechanical coupling have been proposed. In Ref.[10], the authors have developed a model to describe the coupled motion of a cavity end mirror and cold atoms trapped inside the cavity. It was shown that the atoms can form a distributed Bragg mirror with high reflectivity[11], leading to a superstrong coupling regime for the cavity quantum electrodynamics(CQED) system. In this proposal, the atoms inside the cavity are assuming an initial Bose-Einstein condensate. This requirement for the atom medium was lifted in Ref.[12], where the low-energy collective excitations of the atoms were used to enhance the coupling between the mirror and the cavity field.

While the superstrong coupling regime has not yet been reached in experiments, a regime where the mechanical oscillation frequency is larger than the cavity linewidth has recently been observed[13]. In this quantum regime, it is interesting to explore entanglement shared between mechanical (macroscopic) and microscopic degrees of freedom. This is one of our goals in this paper. In fact, possibilities to entangle the oscillatory motion of a cavity macromirror with the electromagnetic field in the cavity have been explored in various approaches[14, 15], steady state entanglement in the mechanical vibrations of two macroscopic membranes has been studied[16].

In this paper, we consider a Fabry-Pérot (F-P) cavity with a moving end mirror, which is allowed to move un-

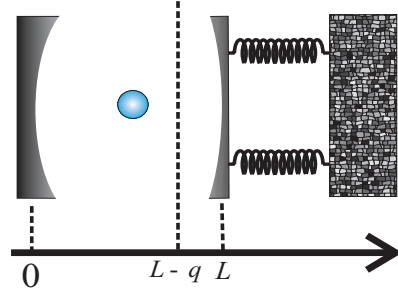


FIG. 1: (Color online) Schematic illustration of an atom in a cavity with a movable end mirror.

der the effect of radiation pressure(see Fig.1). An atom is trapped in the standing-wave light field of the cavity with frequency ω . We shall show that the mechanical vibrations of the end mirror and the atomic center-of-mass(COM) motion can be entangled. Due to the vibration dependent coupling between the atom and the cavity field, the CQED system induces interesting phenomena worth investigating. In contrast to earlier works, our study considers only a single atom inside the cavity, the atomic COM motion and the position dependent atom-to-field couplings may thus be taken into account due to the simplicity of systems involved.

II. MODEL

The system under consideration is shown schematically in Fig.1. An two-level atom with Rabi frequency Ω is trapped in the standing-wave light field of a F-P cavity with one of its end mirrors allowed to move and subject to a harmonic restoring force, q being the displacement of the mirror from its rest position. The cavity length is L and the mass of the movable mirror is m . Although radiation pressure excites several mechanical degrees of freedom, coupling among the different vibrational modes can typically be neglected[15, 16, 17]. We also assume that the electromagnetic field frequency ω follows adiabatically the mirror vibration, hence the frequency of the

cavity field ω is simply parameterized by the mirror position/vibration q . The Hamiltonian of such a system is then[18],

$$H = \hbar\omega a^\dagger a + \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2 + \frac{P^2}{2M} + \frac{\hbar\Omega}{2}\sigma_z - \hbar\xi a^\dagger a q + \hbar g \sin(kQ)(a^\dagger \sigma^- + h.c.), \quad (1)$$

where ω_m denotes the frequency of the mirror vibration, M (m) is the mass of the atom (mirror), and P (p) stands for the momentum of the atom (mirror). The atom-field coupling constant $\hbar g \sin(kQ)$ depends on the atom position Q as well as the vibration q of the mirror through $k = \omega_{eff}/c$, where $\omega_{eff} = \omega - \xi q$ [18], and $\xi = \omega/L$. The most interesting part of the dynamics arises from the coupling term $\hbar g \sin(kQ)(a^\dagger \sigma^- + h.c.)$, describing interactions among the atom, the atomic center-of-mass motion, the cavity field and the vibration of the mirror. We shall analyze its effects in two limiting cases. (a) Slow atomic COM motion: in this regime, we can safely ignore the COM motion of the atom, describing a very cold atom or an atom at a fixed position inside the cavity, and (b) the cavity field is large detuned from the atomic transitions, such that the atomic center-of-mass motion and the vibration of the cavity mirror is slow. In this situation, the center-of-mass motion and the mirror vibration can be included in the analysis. We shall explore the entanglement created among the motion and vibration.

III. SLOW ATOMIC CENTER-OF-MASS MOTION

In this section we study the dynamics of the atom-cavity system when the atomic center-of-mass can be safely ignore. From Eq.(1) we find that the atom-cavity coupling depends on the mirror vibration through $\sin(kQ)$. In general, the dynamics governed by this coupling can not be analytically solved. Hence we shall consider two limiting cases, which may loss some physics with the general coupling, but can shed light on the dynamics with analytical expressions.

A. the case of $k_0 Q_0 = \pi$

For slow atomic center-of-mass motion, we start by assuming that the atom is located at a position Q_0 such that $k_0 Q_0 = \pi$ where $k_0 = \omega/c$. We would like to emphasize that $Q_0 = \pi/k_0$ is a node of the sine mode function of the cavity field, hence this position will not correspond to a potential minimum of the cavity. This indicates that an additional trap has to be introduced to locate the atom. Alternatively, the cavity can be tuned to be blue detuned from the atomic transition and the mean photon number in the cavity can be chosen high enough, such that the atom is stable at that position. We will not specify the trap and only consider an ideal situation where cavity

loss and atomic spontaneous emission are ignored. The Hamiltonian in this case reduces to,

$$H_\pi = \hbar(\omega - \xi q)a^\dagger a + \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2 + \frac{\hbar\Omega}{2}\sigma_z - \hbar g_\pi(a^\dagger \sigma^- + h.c.)q, \quad (2)$$

where $g_\pi = g \sin(\frac{\pi}{L}q)/q$. We consider an example where the amplitude of the mirror vibration is much smaller than the length of the cavity ($q_{max} \ll L$), the coupling constant approximately becomes $g_\pi \simeq g \frac{\pi}{L}$. The coupling between the moving mirror and the CQED system (atom plus cavity field) lead to modifying the atom-cavity interaction. By introducing a transformation $p \rightarrow p' = p$, $q \rightarrow q' = q - \hat{\pi}/m\omega_m^2$, the Hamiltonian H_π can be rewritten as

$$H_\pi = \hbar\omega a^\dagger a + \frac{p'^2}{2m} + \frac{1}{2}m\omega_m^2 q'^2 + \frac{\hbar\Omega}{2}\sigma_z - \frac{\hat{\pi}^2}{2m\omega_m^2}.$$

Here the operator $\hat{\pi}$ was defined as $\hat{\pi} = \hbar\xi a^\dagger a + \hbar g_\pi(a^\dagger \sigma^- + h.c.)$. In the adiabatic limit of the CQED system, namely when the CQED system changes slowly with respect to the fast-varying vibration of the mirror, we can solve the oscillation of the mirror with fixed CQED variables[19], and rewrite the Hamiltonian as,

$$H_\pi^{eff} = \hbar\omega a^\dagger a + \hbar\omega_m(n_m + \frac{1}{2}) + \frac{\hbar\Omega}{2}\sigma_z - \frac{\hat{\pi}^2}{2m\omega_m^2}, \quad (3)$$

where n_m denotes the phonon number of the vibration mirror. In the derivation of Eq.(3), the mirror was assumed in a Fock state $|n_m\rangle$, satisfying $a_m^\dagger a_m |n_m\rangle = n_m |n_m\rangle$ with $a_m = 1/\sqrt{2m\hbar\omega}(m\omega q' + ip')$, and $a_m^\dagger = (a_m)^\dagger$. This treatment is a good approximation when the coupling of mirror vibration to the CQED system does not induce transitions among the Fock states $|n_m\rangle$ with different phonon numbers, namely ($|\psi(t)\rangle$ denotes any state of the CQED system)

$$\left| \langle \psi(t) | \frac{\langle n_m | \hbar g_\pi (a^\dagger \sigma^- + h.c.) q + \hbar \xi a^\dagger a q | m_m \rangle}{\hbar \omega_m (n_m - m_m)} | \psi(t) \rangle \right| \ll 1.$$

Note that $\{|n+1, g\rangle, |n, e\rangle\}$ (n is the photon number in the cavity, while $|e\rangle$ and $|g\rangle$ represent the excited and ground state of the two-level atom, respectively) form an invariant subspace for the effective Hamiltonian Eq.(3), we may diagonalize this Hamiltonian and obtain the following eigenvalues and corresponding eigenstates, respectively,

$$E_{\pm}(n) = \frac{H_{11} + H_{22}}{2} \pm \sqrt{\frac{1}{4}(H_{11} - H_{22})^2 + H_{12}^2} \quad (4)$$

and

$$|+\rangle_n = \begin{pmatrix} \cos \frac{\theta_n}{2} \\ \sin \frac{\theta_n}{2} \end{pmatrix}, |-\rangle_n = \begin{pmatrix} -\sin \frac{\theta_n}{2} \\ \cos \frac{\theta_n}{2} \end{pmatrix}. \quad (5)$$

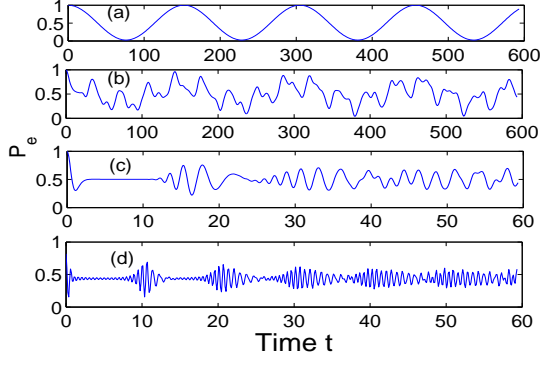


FIG. 2: Population of the atom in its excited state as a function of time. The cavity field initially is prepared in a coherent state $|\alpha\rangle$, while the atom in $|e\rangle$. $m = 10^{-9}$ Kg, $L = 5 \times 10^{-3}$ m, $\omega_m = (2\pi)2.5 \times 10^3$ Hz, $g = \omega = \Omega = (2\pi)4.5 \times 10^6$ Hz. The energy was rescaled by $\hbar\omega_0$ ($\omega_0 = 10^{12}$ Hz here), and the time was rescaled accordingly. (a),(b),(c) and (d) correspond to $\alpha = 0, 1, 3$ and 5 , respectively.

Here θ_n can be determined by $\tan \theta_n = \frac{2H_{12}}{H_{11} - H_{22}}$, and

$$\begin{aligned} H_{11} &= \hbar\omega(n+1) - \frac{\hbar\Omega}{2} - \frac{\hbar^2}{2m\omega_m^2}[\xi^2(n+1)^2 + g_\pi^2(n+1)], \\ H_{22} &= \hbar\omega n + \frac{\hbar\Omega}{2} - \frac{\hbar^2}{2m\omega_m^2}[\xi^2 n^2 + g_\pi^2(n+1)], \\ H_{12} &= -\frac{\hbar^2 g_\pi \xi}{2m\omega_m^2}(2n+1)\sqrt{n+1} = H_{21}. \end{aligned} \quad (6)$$

The dressed states in Eq.(5) are different from that in the Jaynes-Cummings (JC) model: the energy splitting depends on the cavity field (through the photon number) more dramatically than that given by the JC model. As a consequence, the collapse and revivals in the JC model would be modified, leading these interesting features to disappear in this system. This can be found in Fig.2, where we present numerical simulations for the dynamics governed by $H_{\pi/2}^{eff}$. The atom was initially prepared in its excited state $|e\rangle$, while the cavity field was assumed a coherent state $|\alpha\rangle$. Further simulations show that the collapse and revivals are enhanced by large α . This feature can be understood as nonlinear atom-field couplings induced by the vibrating mirror, which speed-up the population transfer between the interval degrees of the atom. Before closing this subsection, we briefly discuss the effect of the atomic spontaneous emission and the cavity decay on the dynamics of the system. The atomic spontaneous emission and the cavity decay may be taken into account by adding an imaginary frequency shift $i\Gamma/2$ to the Rabi frequency Ω , and $i\kappa$ to the cavity frequency ω . The population of the atom in its excited state as a function of time is shown in Fig. 3. Two observations can be made from Fig. 3. (1) The atomic spontaneous emission and the cavity decay make the population a damping function of time (see Fig.3-(a)); (2) The spontaneous emission

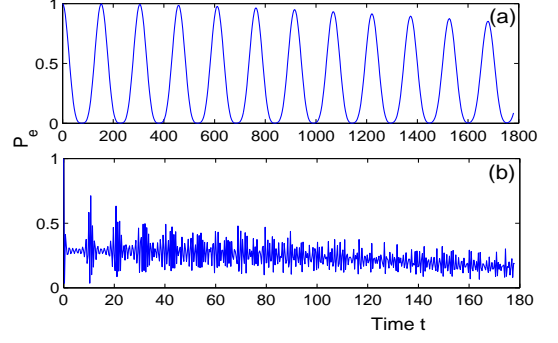


FIG. 3: The same as 2, but with dissipation effects. $\Gamma = 10g$, and $\kappa = 2g$ were chosen for this plot. (a) and (b) are for different α , (a) $\alpha = 0$, (b) $\alpha = 5$.

and the cavity loss spoil the collapse and revival in the dynamics (see Fig.3-(b)).

B. the case of $k_0 Q_0 = \pi/2$

When the atom is placed in a position satisfying $k_0 Q_0 = \pi/2$, the Hamiltonian takes the form,

$$\begin{aligned} H_{\pi/2} &= \hbar(\omega - \xi q)a^\dagger a + \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2 + \frac{\hbar\Omega}{2}\sigma_z \\ &+ \hbar g(a^\dagger \sigma^- + h.c.) - \hbar g_{\frac{\pi}{2}} q^2(a^\dagger \sigma^- + h.c.), \end{aligned} \quad (7)$$

where $g_{\frac{\pi}{2}} \simeq \frac{\pi^2 q}{8L}$. Following the same analysis, we find that the coupling of the mirror to the cavity field induce a nonlinear Kerr effect[23]. The effective Hamiltonian for the CQED system can be expressed as,

$$H_{\pi/2}^{eff} = \hbar\omega a^\dagger a + \frac{\hbar\Omega}{2}\sigma_z + \hbar g(a^\dagger \sigma^- + h.c.) - \frac{\hbar^2 \xi^2}{2m\omega_m^2}(a^\dagger a)^2. \quad (8)$$

This is exactly the Hamiltonian that describes a two-level atom in a cavity filled with a nonlinear Kerr medium.

IV. SLOW MIRROR VIBRATION AND ATOMIC CENTER-OF-MASS MOTION

Optomechanics has attracted considerable attention in recent years not only because of its possible technological use but also because of the theoretical interests in understanding the quantum-classical transition. It is believe that entanglement can act as a bridge between the quantum and classical world. So far, entanglement has been experimentally prepared and manipulated using microscopic quantum systems such as photons, atoms, and ions[20, 21], stationary entanglement between an optical cavity field and a macroscopic vibrating mirror has been theoretically [22], possibility to entangle two macroscopic vibrating mirrors has been explored[16]. It would

be interesting to extend the radiation-pressure-induced entanglement to atomic center-of-mass motion and the vibration of the mirror. From a theoretical point of view, the extension is interesting, because this entanglement is shared between a microscopic (atomic COM motion) and a macroscopic object (vibrating mirror), moreover the atom and the mirror are not directly coupled, but interact with each other through the cavity field.

$$U_{\pm,n} = \frac{2n+1}{2}(\hbar\omega - \hbar\xi q) \pm \sqrt{\hbar^2 g^2 \sin^2(kQ)(n+1) + \frac{1}{4}(\hbar\omega - \hbar\xi q - \hbar\Omega)^2}. \quad (9)$$

Assuming the cavity field and the atomic internal degree of freedom to follow an adiabatic evolution in state $|\phi_{\pm}, n\rangle$, we can write the effective Hamiltonian for the mirror vibration and atomic motion as

$$H_e = \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2 + \frac{P^2}{2M} + U_{\pm,n}. \quad (10)$$

This adiabatic treatment is valid if the coupling of the cavity field to the atomic internal levels is far from resonance, such that the level spacing $|U_{+,n} - U_{-,n}|$ is large, and population transitions between $|\phi_{+}, n\rangle$ and $|\phi_{-}, n\rangle$ can be ignored. Mathematically, this requires

$$\left| \frac{\langle \phi_{+}, n | \frac{\partial}{\partial Q} | \phi_{-}, n \rangle + \langle \phi_{+}, n | \frac{\partial}{\partial q} | \phi_{-}, n \rangle}{U_{+,n} - U_{-,n}} \right| \ll 1,$$

reminiscent of the Born-Oppenheimer approximation. In this situation, $U_{\pm,n}$ can be approximated by,

$$U_{+,n} \simeq -(n+1)\hbar\xi q + \frac{\hbar g^2 k^2 Q^2 (n+1)}{\Delta} - \hbar\xi q \frac{g^2 k^2 Q^2 (n+1)}{\Delta^2}, \quad (11)$$

where $\Delta = \omega - \Omega$. In the remainder of this paper, we shall choose $n = 0$, implying no photon in the cavity while the atom in its excited state. In this case, the effective Hamiltonian Eq.(10) follows,

$$H_e = \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2 + \frac{P^2}{2M} + \frac{\hbar g^2 k^2 Q^2}{\Delta} - \hbar\xi q - \hbar\xi q \frac{g^2 k^2 Q^2}{\Delta^2}. \quad (12)$$

By the canonical quantization, we let p, q, P, Q be operators, which obey the commutation relations, $[q, p] = [Q, P] = i\hbar$, and the others = 0. In terms of creation and annihilation operators, the effective Hamiltonian reads,

$$H_e = \hbar\omega_m(c^\dagger c + \frac{1}{2}) + \hbar\omega'(b^\dagger b + \frac{1}{2}) - \hbar G(c^\dagger + c)(b^\dagger + b)^2, \quad (13)$$

By considering the slow-varying mirror vibration and atomic COM motion, we can solve the coupling of the cavity field to the atomic internal degree of freedom in Eq.(1) first, with fixed q and Q . This approximation is valid when the detuning $(\omega - \Omega)$ is considerably large. We denote $|\phi_{+}, n\rangle$ and $|\phi_{-}, n\rangle$ the eigenstates, the corresponding eigenvalues are given by,

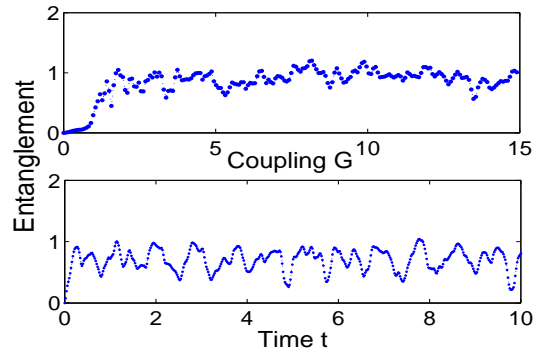


FIG. 4: The entanglement measured by the von Neumann entropy as a function of the coupling constant G (top) and time (bottom). $\omega = (2\pi)6 \times 10^7 \text{ MHz}$, $M = 10^{-26} \text{ Kg}$, $\Delta = 10^3 \text{ Hz}$, $\omega_m = (2\pi)8 \times 10^7 \text{ Hz}$, $m = 5 \times 10^{-9} \text{ Kg}$, $L = 10^{-4} \text{ m}$. The time was set in units of 10^{-4} s , and the coupling constant was plotted in units of MHz. For the top panel $t = 0.1 \text{ ms}$, and in the bottom panel $G = 5 \times 10^3 \text{ Hz}$. The initial state chosen is $|1_c, 1_b\rangle$, where $|n_c, n_b\rangle$ denotes the Fock state of the system.

where the term $\hbar\xi q$ has been ignored, and

$$\omega' \equiv gk\sqrt{\frac{2\hbar}{M\Delta}}, \quad G = \frac{\xi g k \hbar}{4\sqrt{mM\omega\Delta^3}}. \quad (14)$$

Fig.4 shows the entanglement shared between vibration of the mirror and the atomic COM motion, measured by the von Neumann entropy. Remarkably, the entanglement is always not zero when the coupling constant G is considerably large. And to some extent, we can say the entanglement is insensitive to the coupling constant. For the cavity with finite photon number, i.e., $n \neq 0$, the coupling constant G is proportional to $\sqrt{n+1}$, hence it increases the strength of the coupling, however, it does not increase the entanglement shared between the atomic COM motion and the vibration of the mirror, as shown in Fig.4 (see the top panel). For a specific coupling (for example, the effective coupling con-

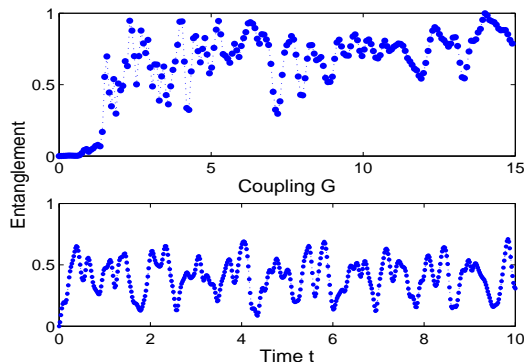


FIG. 5: The same as Fig. 4, but with different initial states. Here the initial state of the vibration of the mirror is chosen to be a thermal state with $\beta = 1/k_B T = 10^{14}$ Hz, while the initial state of the atomic COM is $|1_c\rangle$.

stant $G = 5 \times 10^3$ Hz), the entanglement oscillates with time as shown in the bottom panel of Fig. 4. It is interesting to study when $\omega_m = 2\omega'$. In this case, the Hamiltonian under the rotating-wave approximation is $H_e = \hbar\omega_m(c^\dagger c + \frac{1}{2}) + \hbar\omega'(b^\dagger b + \frac{1}{2}) - \hbar G(c^\dagger b^2 + h.c.)$. The entanglement is a periodic function of time in this case. In order to study the entanglement at finite temperature, we take a thermal distribution of phonon $\rho_T = \sum_{n_b} e^{-\beta n_b \omega_m} |n_b\rangle \langle n_b| / Z$ as an initial state, where $Z = \sum_{n_b} e^{-\beta n_b \omega_m}$. Numerical calculation for the entanglement as a function of time and the coupling constant G is presented in Fig. 5. We find that the thermal effect spoils the creation of entanglement. Nevertheless, the en-

tanglement can still be created between the atomic COM motion and the vibration of the mirror at room temperature. This is possible to observe by recent technology that the radiation-pressure induced correlations between two optical beams was demonstrated [24].

V. CONCLUSION

In conclusion, the dynamics and entanglement of an atom trapped in a cavity with a movable mirror is studied. The key results come from the atom-field-mirror coupling, which depends on the position of the atom and the vibration of the mirror. By manipulating the detuning between the atomic resonance and the cavity field, the coupled system falls into two remarkable regimes: fast CQED regime and slow CQED regime. In the fast CQED regime, we have investigated the dynamics of the CQED system, showing that the interesting feature of collapse and revivals is significantly modified. The entanglement in the atomic center-of-mass motion and the vibration of the mirror has been studied in the slow CQED regime. Interestingly, we found the entanglement is stable against the fluctuation of coupling. The time evolution of the entanglement seems chaotic, except the special case when $2\omega' = \omega_m$.

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